1993 AJHSME Problems/Problem 1

Problem

Which pair of numbers does NOT have a product equal to 36?

$\text{(A)}\ \{-4,-9\}\qquad\text{(B)}\ \{-3,-12\}\qquad\text{(C)}\ \left\{\frac{1}{2},-72\right\}\qquad\text{(D)}\ \{ 1,36\}\qquad\text{(E)}\ \left\{\frac{3}{2},24\right\}$

Solution

A. The ordered pair $-4, -9$ has a product of $-4*-9=36$

B. The ordered pair $-3, -12$ has a product of $-3*-12=36$

C. The ordered pair $1/12, -72$ has a product of -36

D. The ordered pair $1, 36$ has a product of $1*36=36$

E. The ordered pair $3/2, 24$ has a product of $3/2*24=36$

Since C is the only ordered pair which doesn't equal 36, $\boxed{\text{(C)}}$ is the answer.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 2

Problem

When the fraction $\dfrac{49}{84}$ is expressed in simplest form, then the sum of the numerator and the denominator will be

$\text{(A)}\ 11 \qquad \text{(B)}\ 17 \qquad \text{(C)}\ 19 \qquad \text{(D)}\ 33 \qquad \text{(E)}\ 133$

Solution

$$\begin{aligned}\frac{49}{84} &= \frac{7^2}{2^2\cdot 3\cdot 7}\\ &= \frac{7}{2^2\cdot 3}\\ &= \frac{7}{12}.\end{aligned}$$

The sum of the numerator and denominator is $7+12=19\rightarrow \boxed{\text{C}}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

Category: Introductory Algebra Problems

1993 AJHSME Problems/Problem 3

Problem

Which of the following numbers has the largest prime factor?

$\text{(A)}\ 39 \qquad \text{(B)}\ 51 \qquad \text{(C)}\ 77 \qquad \text{(D)}\ 91 \qquad \text{(E)}\ 121$

Solution

A. The prime factors of 39 are 3 and 13. Therefore, the largest prime factor is 13.

B. The prime factors of 51 are 3 and 17. Therefore, the largest prime factor is 17.

C. The prime factors of 77 are 7 and 11. Therefore, the largest prime factor is 11.

D. The prime factors of 91 are 7 and 13. Therefore, the largest prime factor is 13.

E. The only prime factor of 121 is 11. This makes 11 the largest prime factor of this number.

Since we are looking for the number with the largest prime factor, the correct answer would be $\boxed{\text{(B)}}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 4

Problem

$1000 \times 1993 \times 0.1993 \times 10 =$

$\text{(A)}\ 1.993\times 10^3 \qquad \text{(B)}\ 1993.1993 \qquad \text{(C)}\ (199.3)^2 \qquad \text{(D)}\ 1,993,001.993 \qquad \text{(E)}\ (1993)^2$

Solution

$1000 \times 10 = 10^4$
$0.1993 = 1993 \times 10^{-4}$
$1993 \times 1993 \times 10^{-4} \times 10^4 = \boxed{\text{(E)}\ (1993)^2}$

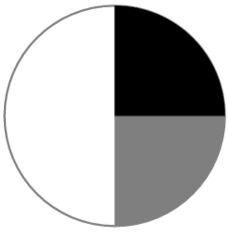
See Also

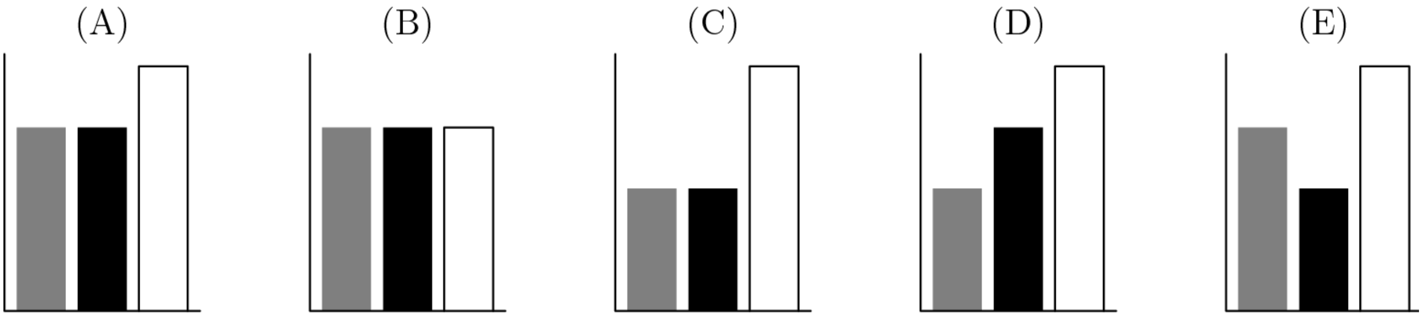
1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 5

Problem

Which one of the following bar graphs could represent the data from the circle graph?





Solution

Since the white portion of the graph is twice as much as the black and gray, its bar in the bar graph will need to be twice as much as the black and gray. Note that the black and gray portions are equal. This cancels choices A, B, D, E. The final answer is now $\boxed{C}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 6

Problem

A can of soup can feed 3 adults or 5 children. If there are 5 cans of soup and 15 children are fed, then how many adults would the remaining soup feed?

$\text{(A)}\ 5 \qquad \text{(B)}\ 6 \qquad \text{(C)}\ 7 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 10$

Solution

A can of soup will feed 5 children so 15 children are feed by 3 cans of soup. Therefore, there are $5-3=2$ cans for adults, so $3\times 2=\boxed{\text{(B)}\ 6}$ adults are fed.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 5	Followed by Problem 7
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 7

Problem

$3^3+3^3+3^3 =$

$\text{(A)}\ 3^4 \qquad \text{(B)}\ 9^3 \qquad \text{(C)}\ 3^9 \qquad \text{(D)}\ 27^3 \qquad \text{(E)}\ 3^{27}$

Solution

$3^3+3^3+3^3=3\times 3^3=\boxed{\text{(A)}\ 3^4}$

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 6	Followed by Problem 8
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 8

Problem

To control her blood pressure, Jill's grandmother takes one half of a pill every other day. If one supply of medicine contains 60 pills, then the supply of medicine would last approximately

$\text{(A)}\ 1\text{ month} \qquad \text{(B)}\ 4\text{ months} \qquad \text{(C)}\ 6\text{ months} \qquad \text{(D)}\ 8\text{ months} \qquad \text{(E)}\ 1\text{ year}$

Solution

If Jill's grandmother takes one half of a pill every other day, she takes a pill every 4 days. Since she has 60 pills, the supply will last $60 \times 4 = 240$ days which is about $\boxed{\text{(D)}\ 8\text{ months}}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 7	Followed by Problem 9
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 9

Problem

Consider the operation $*$ defined by the following table:

$*$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

For example, $3*2=1$. Then $(2*4)*(1*3)=$

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 3 \qquad \text{(D)}\ 4 \qquad \text{(E)}\ 5$

Solution

Using the chart, $(2*4)=3$ and $(1*3)=3$. Therefore, $(2*4)*(1*3)=3*3=\boxed{\text{(D)}\ 4}$.

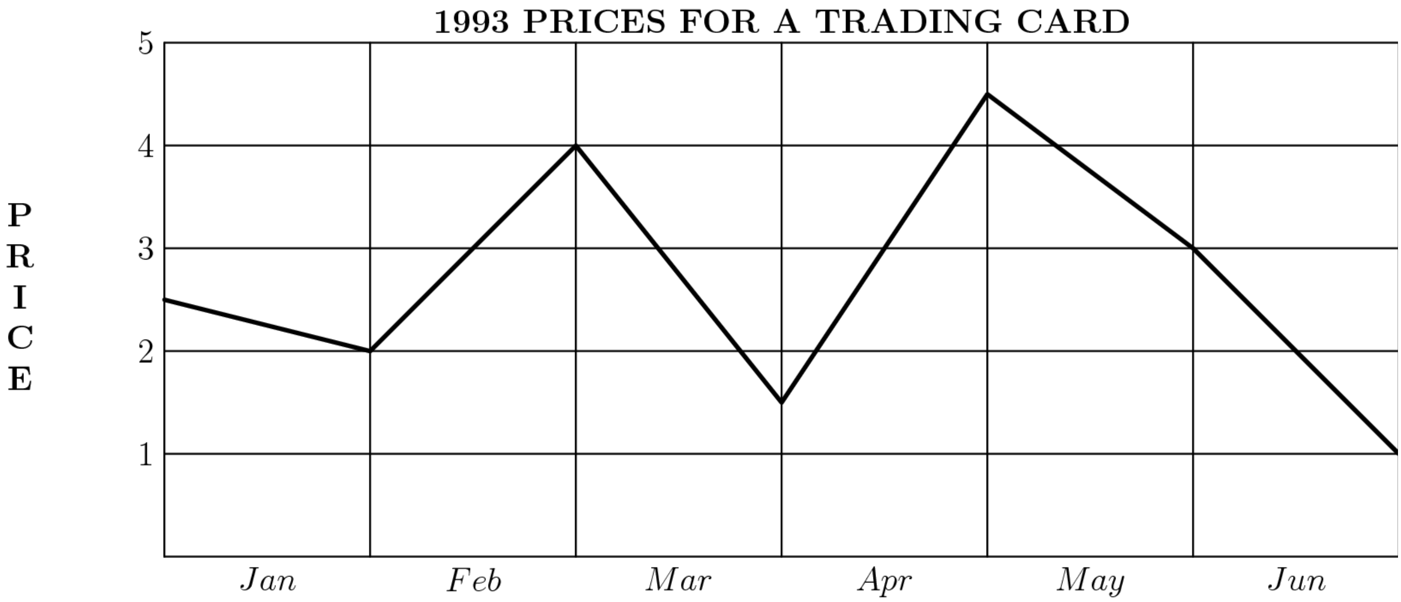
See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 8	Followed by Problem 10
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 10

Problem

This line graph represents the price of a trading card during the first 6 months of 1993.



The greatest monthly drop in price occurred during

$\text{(A)}\ \text{January} \qquad \text{(B)}\ \text{March} \qquad \text{(C)}\ \text{April} \qquad \text{(D)}\ \text{May} \qquad \text{(E)}\ \text{June}$

Solution

The graph shows the following price changes:

Jan : Change:-0.50

Feb : Change:+2.00

Mar : Change:-2.50

Apr : Change:+3.00

May : Change:-0.50

Jun : Change:-2.00

Therefore, $\boxed{\text{(B)}\ \text{March}}$ has the largest price drop.

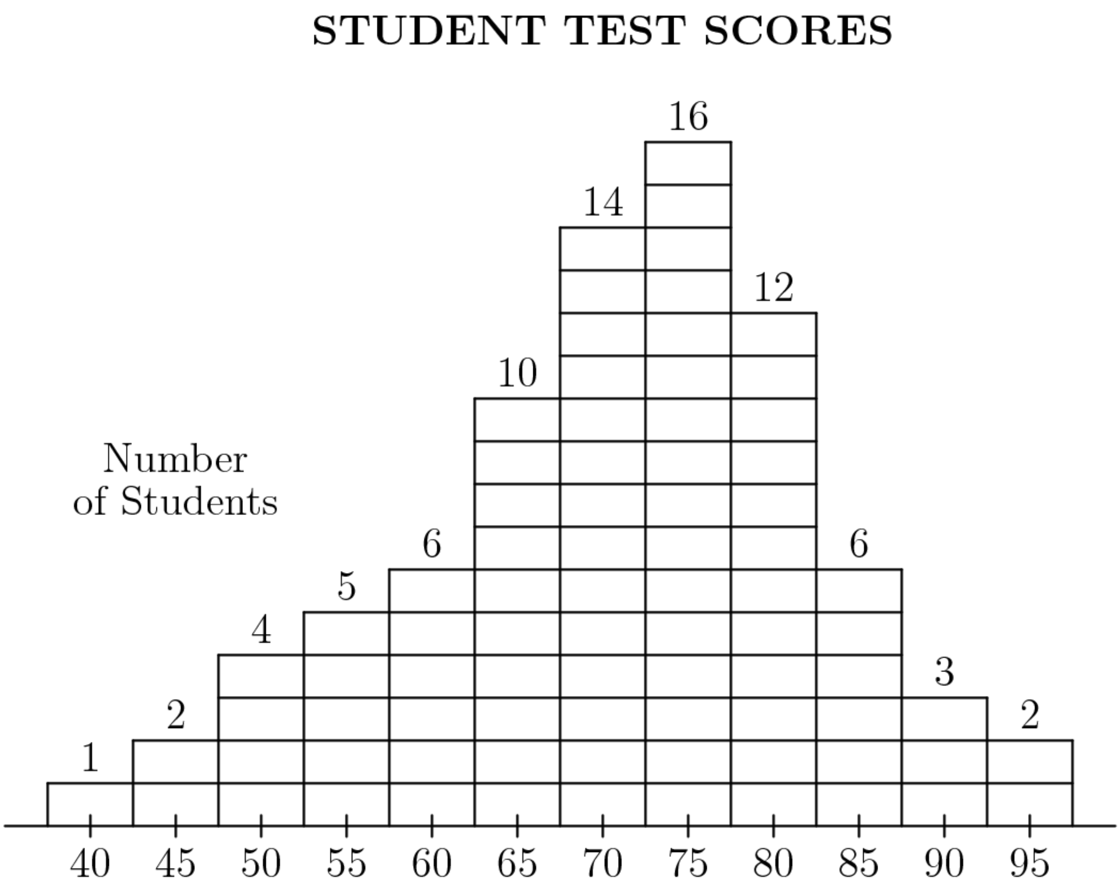
See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 9	Followed by Problem 11
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 11

Problem

Consider this histogram of the scores for 81 students taking a test:



The median is in the interval labeled

$\text{(A)}\ 60 \qquad \text{(B)}\ 65 \qquad \text{(C)}\ 70 \qquad \text{(D)}\ 75 \qquad \text{(E)}\ 80$

Solution

Since 81 students took the test, the median is the score of the 41^{st} student. The five rightmost intervals include $2+3+6+12+16=39$ students, so the 41^{st} one must lie in the next interval, which is $\boxed{\text{(C)}\ 70}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 10	Followed by Problem 12
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All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 12

Problem

If each of the three operation signs, $+$, $-$, $\times$, is used exactly ONCE in one of the blanks in the expression

$$5 \_\_ 4 \_\_ 6 \_\_ 3$$

then the value of the result could equal

$\text{(A)}\ 9 \qquad \text{(B)}\ 10 \qquad \text{(C)}\ 15 \qquad \text{(D)}\ 16 \qquad \text{(E)}\ 19$

Solution

There are a reasonable number of ways to place the operation signs, so guess and check to find that $5-4+6\times 3=\boxed{\text{(E)}\ 19}$.

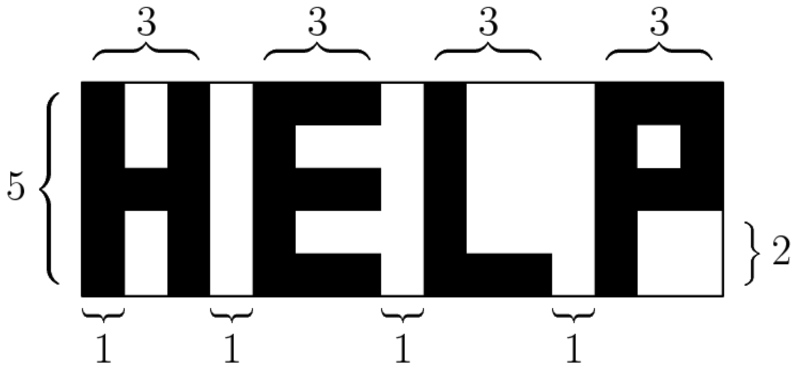
See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 11	Followed by Problem 13
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 13

Problem

The word "HELP" in block letters is painted in black with strokes 1 unit wide on a 5 by 15 rectangular white sign with dimensions as shown. The area of the white portion of the sign, in square units, is



$\text{(A)}\ 30 \qquad \text{(B)}\ 32 \qquad \text{(C)}\ 34 \qquad \text{(D)}\ 36 \qquad \text{(E)}\ 38$

Solution

Count the number of black squares in each letter. H has 11, E has 11, L has 7, and P has 10, giving the number of black squares to be $11+11+7+10=39$. The total number of squares is $(15)(5)=75$ and the number of white squares is $75-39=\boxed{\text{(D)}\ 36}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 12	Followed by Problem 14
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All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 14

Problem

The nine squares in the table shown are to be filled so that every row and every column contains each of the numbers $1, 2, 3$. Then $A + B =$

1		
	2	A
		B

$\text{(A)}\ 2 \qquad \text{(B)}\ 3 \qquad \text{(C)}\ 4 \qquad \text{(D)}\ 5 \qquad \text{(E)}\ 6$

Solution

The square connected both to 1 and 2 cannot be the same as either of them, so must be 3.

1	3	
	2	A
		B

The last square in the top row cannot be either 1 or 3, so it must be 2.

1	3	2
	2	
		B

The other two squares in the rightmost column with A and B cannot be two, so they must be 1 and 3 and therefore have a sum of $1+3=\boxed{\text{(C)}\ 4}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 13	Followed by Problem 15
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All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 15

Problem

The arithmetic mean (average) of four numbers is 85. If the largest of these numbers is 97, then the mean of the remaining three numbers is

$\text{(A)}\ 81.0 \qquad \text{(B)}\ 82.7 \qquad \text{(C)}\ 83.0 \qquad \text{(D)}\ 84.0 \qquad \text{(E)}\ 84.3$

Solution

Say that the four numbers are $a, b, c,$ & 97. Then $\frac{a+b+c+97}{4}=85$. What we are trying to find is $\frac{a+b+c}{3}$. Solving,

$$\frac{a+b+c+97}{4}=85$$

$$a+b+c+97=340$$

$$a+b+c=243$$

$$\frac{a+b+c}{3}=\boxed{\text{(A)}\ 81}$$

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 14	Followed by Problem 16
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All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 16

Problem

$$\frac{1}{1+\frac{1}{2+\frac{1}{3}}} =$$

$\text{(A)}\ \dfrac{1}{6} \qquad \text{(B)}\ \dfrac{3}{10} \qquad \text{(C)}\ \dfrac{7}{10} \qquad \text{(D)}\ \dfrac{5}{6} \qquad \text{(E)}\ \dfrac{10}{3}$

Solution

$$\frac{1}{1+\frac{1}{2+\frac{1}{3}}} = \frac{1}{1+\frac{1}{\frac{7}{3}}} = \frac{1}{1+\frac{3}{7}} = \frac{1}{\frac{10}{7}} = \boxed{\text{(C)}\ \frac{7}{10}}$$

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 15	Followed by Problem 17
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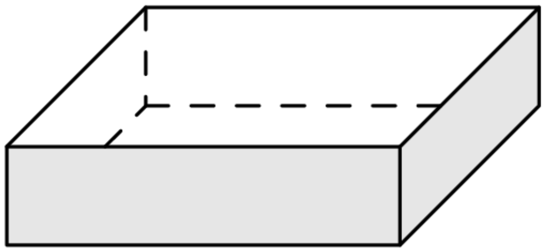
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 17

Problem

Square corners, 5 units on a side, are removed from a 20 unit by 30 unit rectangular sheet of cardboard. The sides are then folded to form an open box. The surface area, in square units, of the interior of the box is



$\text{(A)}\ 300 \qquad \text{(B)}\ 500 \qquad \text{(C)}\ 550 \qquad \text{(D)}\ 600 \qquad \text{(E)}\ 1000$

Solution

If the sides of the open box are folded down so that a flat sheet with four corners cur out remains, then the revealed surface would have the same area as the interior of the box. This is equal to the area of the four corners subtracted from the area of the original sheet, which is

$(20)(30)-4(5)(5)=600-100=\boxed{\text{(B)}\ 500}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 16	Followed by Problem 18
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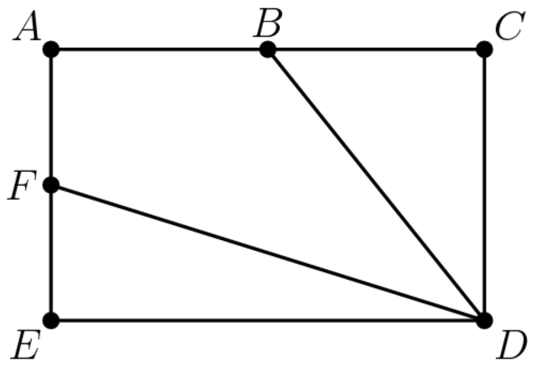
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 18

Problem

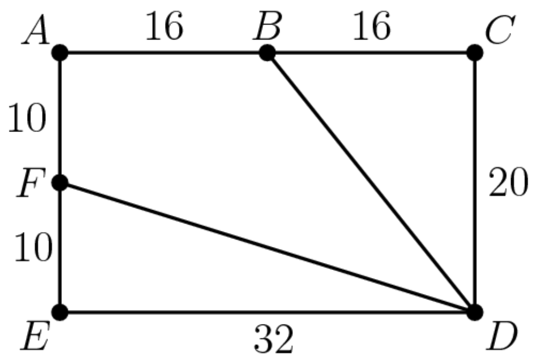
The rectangle shown has length $AC = 32$, width $AE = 20$, and B and F are midpoints of $\overline{AC}$ and $\overline{AE}$, respectively. The area of quadrilateral $ABDF$ is



$\text{(A)}\ 320 \qquad \text{(B)}\ 325 \qquad \text{(C)}\ 330 \qquad \text{(D)}\ 335 \qquad \text{(E)}\ 340$

Solution

The area of the quadrilateral $ABDF$ is equal to the areas of the two right triangles $\triangle BCD$ and $\triangle EFD$ subtracted from the area of the rectangle $ABCD$. Because B and F are midpoints, we know the dimensions of the two right triangles.



$$(20)(32)-\frac{(16)(20)}{2}-\frac{(10)(32)}{2}=640-160-160=\boxed{\text{(A)}\ 320}$$

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 17	Followed by Problem 19
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 19

Problem

$(1901+1902+1903+\cdots+1993)-(101+102+103+\cdots+193)=$

$\text{(A)}\ 167,400 \qquad \text{(B)}\ 172,050 \qquad \text{(C)}\ 181,071 \qquad \text{(D)}\ 199,300 \qquad \text{(E)}\ 362,142$

Solution

We see that $1901=1800+101$, $1902=1800+102$, etc. Each term in the first set of numbers is 1800 more than the corresponding term in the second set; Because there are 93 terms in the first set, the expression can be paired up as follows and simplified:

$$(1901-101)+(1902-102)+(1903-103)+\cdots+(1993-193)=1800+1800+1800+\cdots+1800=(1800)(93)=\boxed{\text{(A)}\ 167,400}$$

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 18	Followed by Problem 20
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All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 20

Problem

When $10^{93}-93$ is expressed as a single whole number, the sum of the digits is

$\text{(A)}\ 10 \qquad \text{(B)}\ 93 \qquad \text{(C)}\ 819 \qquad \text{(D)}\ 826 \qquad \text{(E)}\ 833$

Solution

$$10^2-93=7$$
$$10^3-93=907$$
$$10^4-93=9907$$

This can be generalized into 10^n-93 is equal is $n-2$ nines followed by the digits 07. Then $10^{93}-93$ is equal to 91 nines followed by 07. The sum of the digits is equal to $9(91)+7=819+7=\boxed{\text{(D)}\ 826}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 19	Followed by Problem 21
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All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 21

Contents

Problem

If the length of a rectangle is increased by 20% and its width is increased by 50%, then the area is increased by

$\text{(A)}\ 10\% \qquad \text{(B)}\ 30\% \qquad \text{(C)}\ 70\% \qquad \text{(D)}\ 80\% \qquad \text{(E)}\ 100\%$

Solution

If a rectangle had dimensions 10×10 and area 100, then its new dimensions would be 12×15 and area 180. The area is increased by $180 - 100 = 80$ or $80/100 = \boxed{\text{(D)}\ 80\%}$.

Alternate Solution

Let the dimensions of the rectangle be $x \times y$. This rectangle has area xy. The new dimensions would be $1.2x \times 1.5y$, so the area is $= 1.8xy$, which is $\boxed{80\%}$ more than the original area.

See Also

1993 AJHSME Problems/Problem 22

Problem

Pat Peano has plenty of 0's, 1's, 3's, 4's, 5's, 6's, 7's, 8's and 9's, but he has only twenty-two 2's. How far can he number the pages of his scrapbook with these digits?

$\text{(A)}\ 22 \qquad \text{(B)}\ 99 \qquad \text{(C)}\ 112 \qquad \text{(D)}\ 119 \qquad \text{(E)}\ 199$

Solution

There is 1 two in the one-digit numbers.

The number of two-digit numbers with a two in the tens place is 10 and the number with a two in the ones place is 9. Thus the digit two is used $10+9=19$ times for the two digit numbers.

Now, Pat Peano only has $22-1-19=2$ remaining twos. The last numbers with a two that he can write are 102 and 112. He can continue numbering the last couple pages without a two until 120, with the last number he writes being $\boxed{\text{(D)}\ 119}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 21	Followed by Problem 23
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1993 AJHSME Problems/Problem 23

Problem

Five runners, P, Q, R, S, T, have a race, and P beats Q, P beats R, Q beats S, and T finishes after P and before Q. Who could NOT have finished third in the race?

$\text{(A)}\ P\text{ and }Q \qquad \text{(B)}\ P\text{ and }R \qquad \text{(C)}\ P\text{ and }S \qquad \text{(D)}\ P\text{ and }T \qquad \text{(E)}\ P,S\text{ and }T$

Solution

First, note that P must beat Q, R, T, and by transitivity, S. Thus P is in first, and not in 3rd. Similarly, S is beaten by P, Q, and by transitivity, T, so S is in fourth or fifth, and not in third. All of the others can be in third, as all of the following sequences show. Each follows all of the assumptions of the problem, and they are in order from first to last: PTQRS, PTRQS, PRTQS. Thus the answer is $\boxed{\text{(C)}\ P\text{ and }S}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 22	Followed by Problem 24
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1993 AJHSME Problems/Problem 24

Contents

Problem

What number is directly above 142 in this array of numbers?

$$\begin{array}{cccccc}& & & 1 & &\\ & & 2 & 3 & 4 &\\ & 5 & 6 & 7 & 8 & 9\\ 10 & 11 & 12 & \cdots & &\end{array}$$

$\text{(A)}\ 99 \qquad \text{(B)}\ 119 \qquad \text{(C)}\ 120 \qquad \text{(D)}\ 121 \qquad \text{(E)}\ 122$

Solution

Solution 1

Notice that a number in row k is $2k$ less than the number directly below it. For example, 5, which is in row 3, is $(2)(3)=6$ less than the number below it, 11.

From row 1 to row k, there are $k\left(\frac{1+(-1+2k)}{2}\right)=k^2$ numbers in those k rows. Because there are $12^2=144$ numbers up to the 12th row, 142 is in the k^{th} row. The number directly above is in the 11th row, and is 22 less than 142. Thus the number directly above 142 is $142-22=\boxed{\text{(C)}\ 120}$.

Solution 2

Writing a couple more rows, the last number in each row ends in a perfect square. Thus 142 is two left from the last number in its row, 144. One left and one up from 144 is the last number of its row, also a perfect square, and is 121. This is one right and one up from 142, so the number directly above 142 is one less than 121, or $\boxed{\text{(C)}\ 120}$.

See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 23	Followed by Problem 25
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1993 AJHSME Problems/Problem 25

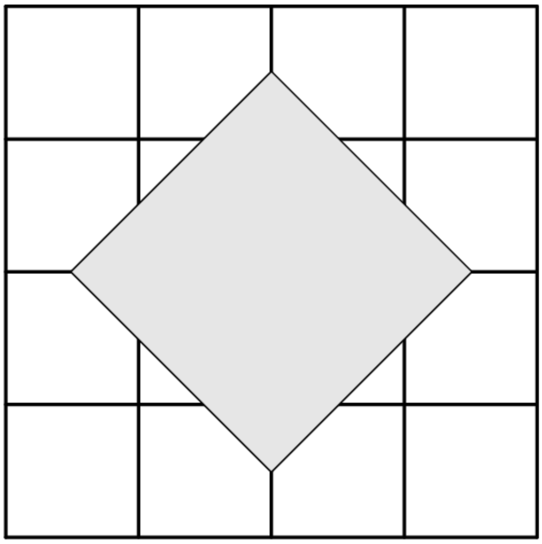
Problem

A checkerboard consists of one-inch squares. A square card, 1.5 inches on a side, is placed on the board so that it covers part or all of the area of each of n squares. The maximum possible value of n is

$\text{(A)}\ 4\text{ or }5 \qquad \text{(B)}\ 6\text{ or }7 \qquad \text{(C)}\ 8\text{ or }9 \qquad \text{(D)}\ 10\text{ or }11 \qquad \text{(E)}\ 12\text{ or more}$

Solution

Using Pythagorean Theorem, the diagonal of the square $\sqrt{(1.5)^2+(1.5)^2} = \sqrt{4.5} > 2$. Because this is longer than 2, the length of the sides of two adjacent squares, the card can be placed like so, covering 12 squares. $\rightarrow \boxed{\text{(E)}\ 12\text{ or more}}$.



See Also

1993 AJHSME (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1993))	
Preceded by Problem 24	Followed by Last Problem
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